

On the Use of Homogeneous Transformations to Map Human Hand Movements onto Robotic Hands

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Abstract—Replicating the human hand capabilities is a great challenge in telemanipulation as well as in autonomous grasping and manipulation. One of the main issues is the difference between human and robotic hands in terms of kinematic structure, which does not allow a direct correlation of the joints. We recently proposed an object-based mapping algorithm able to replicate on several robotic hand models the human hand synergies. In such approach the virtual object shapes were a-priori defined (e.g. a sphere or an ellipsoid) and the transformation was represented as the composition of a rigid body motion and a scale variation. In this work, we introduce a generalization of the object-based mapping that overcomes the definition of a shape for the virtual object. We consider only a set of reference points on the hands. We estimate a homogeneous transformation matrix that represents how the human hand motion changes its reference point positions. The same transformation is then imposed to the reference points on the robotic hand and the joints values obtained through a kinematic inversion technique. The mapping approach is suitable also for telemanipulation scenarios where the hand joint motions are combined with a wrist displacement.

I. INTRODUCTION

Robotic hands present a high variability of kinematic structures, actuation and control systems. They differ in the number of fingers, in the number of Degrees of Freedom (DoFs) per finger, in the type of joints and actuators, etc. [1]. In most of the telemanipulation scenarios, however, the motion of such a heterogeneous set of devices is related to the motion of a unique complex kinematic structure: the human hand. This has led to the development of several mapping strategies that strongly depend on the robotic hand structures. Examples of these approaches are the fingertip mapping [2], the pose mapping [3] and the joint-to-joint mapping [4]. The main drawbacks of these methods are mainly the lack of generality and the need of empirical or heuristic considerations to define the correspondence between human and robotic hands.

To overcome such limits, we presented a mapping defined in the task space and mediated by a virtual object. The method was detailed in [5] considering a sphere as virtual object and generalized in [6], considering an ellipsoid to extend the possible transformations that can be imposed. The

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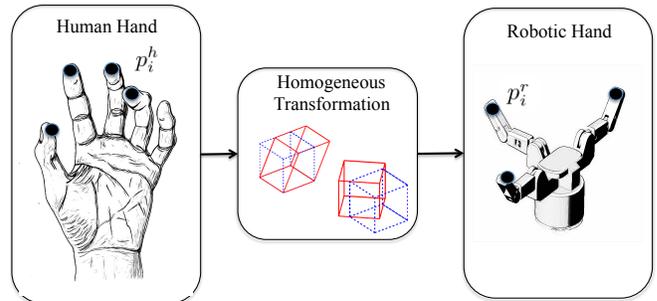


Fig. 1: The mapping framework. The motion of the human hand is captured by a set of reference points (p_i^h) which are used to compute a common homogeneous transformation. The homogeneous transformation is also applied to a set of reference points placed on the robotic hand (p_i^r) and the new configuration computed using inverse kinematic techniques.

object-based mapping is obtained considering two virtual objects, respectively on the human and on the robotic hand. They are computed considering the minimum volume object containing reference points suitably defined, placed on the respective hands. A configuration variation of the human hand induces a motion and a deformation of the virtual object. We impose that the object defined on the robotic hand moves and deforms according to that defined on the human hand. The mapped motion of the robotic hand is then obtained through pseudo-inversion techniques. This mapping procedure, was adopted to map human hand synergies [7], onto hands with very dissimilar kinematics [8]. However, the definition of a shape for the virtual object implies that the method obtains better results if the manipulated and the virtual object are similar. Moreover, shearing deformation cannot be reproduced and only in-hand manipulation has been tested so far, since the few available parameters are not sufficient to describe both joints and wrist motions.

In this paper we introduce a new solution which overcomes the definition of a virtual object shape and allows replicating the motion of the human hand in a telemanipulation scenario where also wrist motion is considered. The mapping is based on the definition of a set of reference points, both on the human and on the robotic hand: the reference points on the human hand are necessary to evaluate the transformation produced by the hand motion, the points on the robotic hands are necessary to map such transformation on the robotic hand (Fig. 1). A configuration change on the human hand causes a transformation of the reference point positions, which can be generally represented by a six-dimensional displacement and/or a non rigid deformation. In this paper, we assume that this transformation can be represented as a linear trans-

formation, estimated from the displacement of the reference points. The same linear transformation is then imposed to the robotic hand reference points and the hand joint displacement is consequently defined by solving its inverse kinematics. A linear transformation matrix can be, in general, decomposed as the combination of different elementary transformations, e.g., translation and rotations, scale variations and shear deformations. Such decomposition is adopted in the proposed mapping procedure to separately reproduce the contribution in terms of internal forces [9], which are paramount for grasp control, and in terms of the rigid body displacement imposed by the hand on the manipulated object [10]. We simulated a possible teleoperation scenario to test the applicability and effectiveness of the proposed approach. In the teleoperation task both hand and wrist motion are involved.

The paper is organized as it follows. Section II summarizes the properties of homogeneous transformations and recalls the definition of primitive transformations. Section III describes the procedure to map human hand movements to robotic hands, based on homogeneous transformations, while Section IV shows how the proposed procedure works with some numerical tests, in which a robotic hand with a non anthropomorphic structure is simulated. Finally Section V provides some concluding considerations and describes further developments of the work.

II. THE HOMOGENEOUS TRANSFORMATION PROPERTIES

The mapping procedure is based on the analysis of the transformation of a set of reference points on the human hand during hand motion. In the paper we assume that the transformation is linear, so, indicating with p_i and p_f the coordinates of a generic reference point in its initial and final configuration, respectively, we have that

$$p_f = Ap_i + b,$$

where A is a 3×3 matrix representing the linear map and b is a three-dimensional vector representing the translation in the transformation. Introducing the augmented matrix and vector notation, it is possible to represent both the translation and the linear map using a single matrix multiplication, i.e.

$$\hat{p}_f = T\hat{p}_i,$$

where $\hat{p}_{i,f} = \begin{bmatrix} p_{i,f}^T & 1 \end{bmatrix}^T$ and

$$T = \begin{bmatrix} A & p \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (1)$$

Homogeneous 4×4 matrices are widely used in 3D computer graphics system to represent solid bodies transformations [11]. Homogeneous transformations are able to represent all the transformations required to move an object and visualize it: translation, rotation, scale, shear, and perspective. Any number of transformations can be multiplied to form a composite matrix. Transformation matrices are widely adopted also in continuum mechanics to describe material displacements and strains, and methods to decompose from the deformation gradient the contribution of rigid body motion and non rigid deformation are available in the literature, see for instance [12], [13].

Rigid body motions are particular types of transformation that preserve the distance between points and the angles between vectors. They can be represented as the combination of a rotation, defined by the rotation matrix $R \in SO(3)$, and a translation motion, defined by the vector $p \in \mathbb{R}^3$. $SO(3)$ (special orthogonal) is the set of all the 3×3 orthogonal matrices with determinant equal to 1 [14]. The corresponding homogeneous matrix can be expressed as shown in Fig. 2-a). T is in this case a representation of a generic element of the $SE(3)$ group (special Euclidean).

Homogeneous matrices can be adopted also to describe non rigid transformations: isotropic transformations, which modifies the object size by a scaling factor α , without moving it; non isotropic transformations, which modify the object size by scaling factors $[\alpha, \beta, \gamma]$, in the x, y, z directions respectively, and shear transformations, that displaces each point in fixed direction, by an amount proportional to its signed distance from a line that is parallel to that direction. A generic non rigid transformation is qualitatively represented in Fig. 2-b). In this study we do not consider perspective transformations for the sake of simplicity. These basic homogeneous transformations are usually referred to as primitive transformations. Each of them can be represented with a more meaningful and concise representation: a scalar for the isotropic transformation, a vector for translation, 3D scaling and shear, a quaternion for rotations. The recover of the concise form from the primitive transformation matrix is straightforward, but, on the other hand, once primitives have been multiplied into a composite matrix, the recovery of the primitive is not direct in general [15].

Different procedures to decompose a generic 4×4 matrix into a series of primitive transformations are available in the literature. In this paper we consider the extraction of the rigid and non rigid motions contributions from a generic linear transformation matrix. Consider, for instance, the human hand/arm system moving along a trajectory while the hand is changing the grasp forces exerted onto an object. In this case a large rigid arm displacement is coupled with a smaller non rigid deformation. The displacements of the reference points on the human hand however contains both the contributions. So that, in the mapping procedure, when the arm and wrist is involved in the motion, we propose to extract the rigid part of the motion from the complete transformation matrix and to reproduce it with the robotic arm, while the non rigid contribution to the reference point configuration variation is reproduced acting on the robotic hand fingers. This decomposition will be better explained in the numerical experiment proposed in Sec. IV.

To consider rigid and non-rigid contribution, we need to express the transformation matrix as follows

$$T = T_{def}T_{rb}, \quad (2)$$

where $T_{rb} = T_{tr}T_{rot}$ represents the rigid part of the displacement, composed of a rotation and a translation, and T_{def} takes into account the non rigid deformation, as shown in Fig. 2-a) and b) respectively. The extraction of the translation part of the rigid body motion from the starting matrix T is straightforward considering eq. (1). The matrix A in eq. (1) can be written, with the polar decomposition, as the product

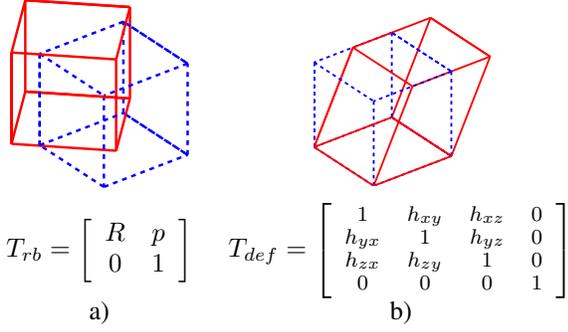


Fig. 2: Examples of linear transformations applied to a cube and corresponding homogeneous matrices: a) rigid body motion, b) shear and scale transformation.

$$A = UR \quad (3)$$

in which R is orthogonal and U is an Hermitian semi-positive matrix, $R \in SO(3)$ represents a rigid rotation, while U takes into account the non rigid deformation [16].

III. MAPPING PROCEDURE BASED ON HOMOGENEOUS TRANSFORMATION

In this work, a model of the human hand is considered as reference hand. The definition of the following reference frames is necessary to identify hand motion. Let $\{N^w\}$ be an inertial reference frame, adopted to describe hand/wrist motion. Let $\{N^h\}$ indicate a frame on the human hand palm. The configuration of $\{N^h\}$ reference frame with respect to the inertial one depends on arm motion, and is described by the homogeneous transformation matrix T_h , which depends on six parameters, namely the coordinates of $\{N^h\}$ origin and the relative orientation between the frames, described for instance by Euler angles. Let $p_w \in \mathbb{R}^6$ be a vector containing $\{N^h\}$ position and orientation with respect to $\{N^w\}$.

Various kinematic models of the human hand are available in the literature, we chose a 20 DoFs model, in which each finger has four DoFs [17]. Let us indicate with $q^h \in \mathbb{R}^{n_{q_h}}$, $n_{q_h} = 20$, the generic configuration of hand joints.

The number and position of the reference points on the human hand, n_h , can be arbitrarily set. Reference points can be placed on the fingertips, in the intermediate phalanges, in correspondence of the joint axis, etc. The fingertips represent a natural choice for the reference points, since they are at the end of the kinematic chains that define the fingers, so their configurations depend on all the joints [5].

Let us indicate with p_k , with $k = 1, \dots, n_h$ the reference points on the human hand. The vector $p_{k,c}^h \in \mathbb{R}^3$ represents the coordinates of the generic reference point p_k with respect to $\{N^h\}$ when the hand assumes a configuration \mathcal{C} and the joint values are q_c^h . Let furthermore us indicate with $\hat{p}_{k,c}^h \in \mathbb{R}^4$ the corresponding augmented vectors, adopted to represent affine transformations, i.e. $\hat{p}_{k,c}^h = [p_{k,c}^h \quad 1]^T$. Finally, let us indicate with $p_c^h \in \mathbb{R}^{3n_h}$ a vector containing the coordinates of all the reference points in the generic configuration \mathcal{C} .

Let p_i^h denote the initial position of the reference points on the human hand. Their position is a function of hand initial configuration vector q_i^h and the wrist initial configuration $p_{i,w}^h$, and can be evaluated by the direct kinematic analysis of the hand, i.e.

$$p_i^h = f_k(q_i^h, p_{i,w}^h). \quad (4)$$

Let us then assume that, starting from this initial configuration, the hand and the wrist are moved, let q_f^h and $p_{f,w}^h$ be the final hand joint and wrist configurations. The reference point positions on the human hand vary according to hand and wrist kinematics, i.e. $p_f^h = f_k(q_f^h, p_{f,w}^h)$ [18].

We assume that the configuration variation of the reference points from p_i^h to p_f^h can be represented as a linear transformation, i. e. for each point p_k , with $k = 1, \dots, n_h$, the following linear relationship holds

$$\hat{p}_{k,f}^h = T \hat{p}_{k,i}^h. \quad (5)$$

Given the initial and final reference point configurations $\hat{p}_{k,i}^h$ and $\hat{p}_{k,f}^h$, we can evaluate the linear transformation T in eq. (5) by solving the following linear system

$$\hat{p}_{k,f}^h = Mt, \quad (6)$$

where $t \in \mathbb{R}^{12}$ contains the components of the linear transformation T , i.e.

$$T = \begin{bmatrix} t_1 & t_2 & t_3 & t_4 \\ t_5 & t_6 & t_7 & t_8 \\ t_9 & t_{10} & t_{11} & t_{12} \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

and the system matrix $M \in \mathbb{R}^{3n_h \times 12}$ is defined as

$$M = \begin{bmatrix} M_1 \\ \dots \\ M_{n_h} \end{bmatrix},$$

in which the generic matrix $M_k \in \mathbb{R}^{3 \times 12}$ is given by

$$M_k = \begin{bmatrix} \hat{p}_{k,i}^h \text{ }^T & 0_{1,4} & 0_{1,4} \\ 0_{1,4} & \hat{p}_{k,i}^h \text{ }^T & 0_{1,4} \\ 0_{1,4} & 0_{1,4} & \hat{p}_{k,i}^h \text{ }^T \end{bmatrix}.$$

As already mentioned in Sec. II, T matrix can then be decomposed as the product between a rigid body transformation matrix T_{rb} and a non rigid transformation T_{def} .

The idea behind the proposed mapping procedure is then to reproduce, on the reference points defined on the robotic hand, the same linear transformation computed on the human hand. Note that the homogeneous transformation matrix T obtained by solving the linear system in eq. (6) depends on the human hand and wrist configuration variation, and also on the initial configuration of reference points p_i^h .

Let us indicate with $\{N^{w,r}\}$ an inertial reference frame, adopted to describe robotic hand and arm motion, and $\{N^r\}$ a reference frame on the robotic hand palm, let $p_{w,r} \in \mathbb{R}^6$ be a vector describing the position and orientation of frame $\{N^r\}$ with respect to $\{N^{w,r}\}$, and let $q_c^r \in \mathbb{R}^{n_{q_r}}$ indicate the robotic hand joint vector. In general, since the robotic and human hands have a different kinematic structure, $n_{q_r} \neq n_{q_h}$.

A set of reference points are defined also on the robotic hand, indicated with p_s^r , with $s = 1, \dots, n_r$. In general, n_h and n_r are not related and $n_h \neq n_r$.

In the initial reference configuration the coordinates of the reference points on the robotic hand are defined by the vectors $p_{s,i}^r$, that can be collected in the vector $p_i^r \in \mathbb{R}^{3n_r}$. The final configuration of these points, according to the above defined linear transformation, can be evaluated as the composition of two motions

$$\hat{p}_{s,f}^r = T_{def} T_{rb} \hat{p}_{s,i}^r = T_{def} \hat{p}_{s,rb}^r. \quad (7)$$

The reference point configurations after the rigid transformation can be collected in the vector $p_{rb}^r \in \mathbb{R}^{3n_r}$, while $p_f^r \in \mathbb{R}^{3n_r}$ contains the final reference point configurations. The displacement vector Δp^r due to the non rigid part of the transformation is thus defined as

$$\Delta p^r = p_f^r - p_{rb}^r.$$

This displacement has to be reproduced by modifying the robotic hand joint values, according to robotic hand inverse kinematics. If the displacement Δp^r is sufficiently small, the linear approximation of the kinematics of the robot can be considered. Consequently, the displacement that has to be imposed to the robotic hand joints, can be evaluated as

$$\Delta q^r = J_r^\# \Delta p^r + N_{J_r} \xi \quad (8)$$

where J_r is the robotic hand Jacobian matrix, the index $\#$ denotes a generic pseudoinverse, N_{J_r} is a basis of J_r nullspace and ξ is a vector parametrizing the homogeneous part of the inverse differential kinematics problem and managing the presence of eventual redundant hand DoF [14]. Robotic hand joint variation Δq^r is the displacement that has to be imposed to the robotic hand joints in order to obtain, on the robotic hand reference points, the same linear transformation of the reference points on the human hand. If the mapping is applied between two hand/arm systems, the rigid body component of the motion, given by

$$\Delta p_{rb}^r = p_{rb}^r - p_i^r$$

can be exerted by the wrist/arm motion while the non-rigid deformation is related to the hand joints.

The main steps of the mapping algorithm are summarized in Fig. 3.

IV. NUMERICAL EXPERIMENT: TELEOPERATING A THREE FINGERED ROBOTIC HANDS

This set of numerical simulations is aimed at mapping on a robotic hand, a task in which the human hand is manipulating a cubic object. The experiments were performed using and adapting the functions available in SynGrasp, a Matlab Toolbox for the simulation and the analysis of grasping with several hand models [19]. In the assigned task, the human wrist moves along a given trajectory and, at the same time, the first hand synergy is activated. Postural synergies represent a way to simplify human hand structure, reducing the number of DoFs necessary to define its posture [20]. The synergy idea has been brought to robotics, to reduce the

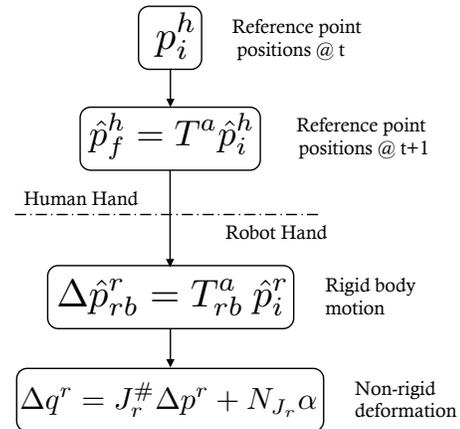


Fig. 3: The mapping algorithm. The motion of the reference points in the human hand between instant t and $t + 1$ is represented with the homogeneous transformation T^a , where T^a collects in a diagonal matrix the T matrix defined for each point. The rigid body transformation matrix T_{rb}^a is used to compute the rigid body contribution, while the non-rigid transformation is obtained acting on robotic hand joints q^r .

number of inputs necessary to actuate a robotic hand, thus simplifying their mechanical and control structure [21], [7]. The synergy activation in this case produces both a variation of the contact forces and a displacement of the object center, which can be evaluated using the procedure discussed in [22]. These wrist and in-hand motions were mapped on a robotic system represented by a three fingered hand resembling the Barrett Hand but with eight actuated joints (two fingers with three joints and one with two), and a six DoF arm. Both the hands were grasping the same object, a cube with side 50mm. The two hands started from a given grasp obtained through the grasp planner available in SynGrasp, which considers a procedure similar to that described in [23]. As reference points for the mapping we assumed the contact points of the hands with the object. In Fig. 4-a) human and robotic hand configurations are sketched, and in Fig. 4-b) contact points are shown. The grasp planner provided seven contact points on the human hand and three contact points on the robotic hand.

We considered a generic trajectory for the human wrist represented by a cosine arc whose length was 470 mm in the x direction and whose height in the z direction was 150 mm with respect to the $\{N^w\}$ reference frame. While the human wrist is following this trajectory, hand finger joint reference values are varied along the first hand synergy. The trajectory was sampled in a series of steps. For each step, human hand and arm motion were mapped on the robotic system with the proposed mapping approach.

Fig. 5-a) shows the human hand trajectory. The blue line represents object center displacement. Fig. 5-b) shows the resulting motion on the robotic hand. The red curve represents robotic hand object center trajectory during the simulation. As it can be seen, the robotic hand is able to follow the human hand trajectory, even if it is the combination of a

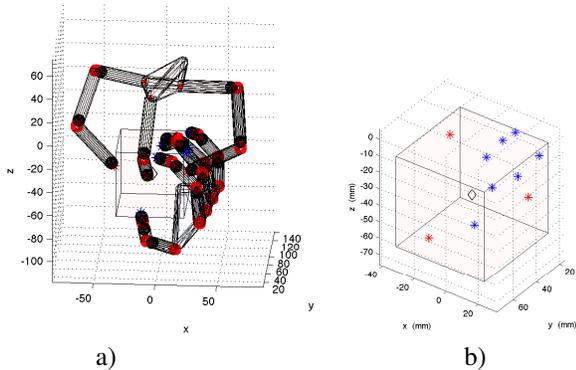


Fig. 4: a) Human hand and three-fingered robotic hand grasping a cubic object: initial configuration b) object center (black dot), contact points on the human hand (blue dots) and on the robotic hand (red dots).

generic six-dimensional wrist displacement with an in-hand motion.

Fig. 6 shows, for the first sampling step, the direction of the object center displacement produced by the activation of the hand synergies, i.e. without considering wrist motion, and the corresponding variations of the contact forces, evaluated according to [22]. While the object motion directions are quite comparable, a simple comparison between contact force variations is not possible, due to the different positions and number of contact points between the human and robotic hand.

Finally, Fig.7 shows the sensitivity of the proposed mapping procedure with respect to some operative parameters. The upper diagram shows the effect of the applied synergies. The final reference values of the human hand joints were evaluated as $q_f^h = q_i^h + \gamma S \Delta \sigma$, with γ varying from 0.1 and 1, S representing the synergy matrix as defined in [21] and $\Delta \sigma = [1 \ 0 \ 0 \ \dots]^T$ for the first synergy, $\Delta \sigma = [0 \ 1 \ 0 \ \dots]^T$ for the second one, etc.. As it can be seen, the sensitivity of γ parameter on the trajectory error, defined as the distance between object centers at the end of trajectory execution, is quite evident. The sensitivity is furthermore different for the different synergies. This effect is due to the different kinematics between human and robotic hand. The three fingered robotic hand, due to its kinematic constraints, is not able to fully reproduce the object in-hand motion produced by the human hand. The lower diagram shows the sensitivity of the trajectory error on the size of trajectory discretization step. In this case the sensitivity is quite lower, i.e. the proposed mapping procedure is quite robust with respect to the length of trajectory discretization step.

V. CONCLUSIONS

The complex and different structures that characterize robotic hands requires methods to unify their control. There are applications, e.g. telemanipulation or learning by demon-

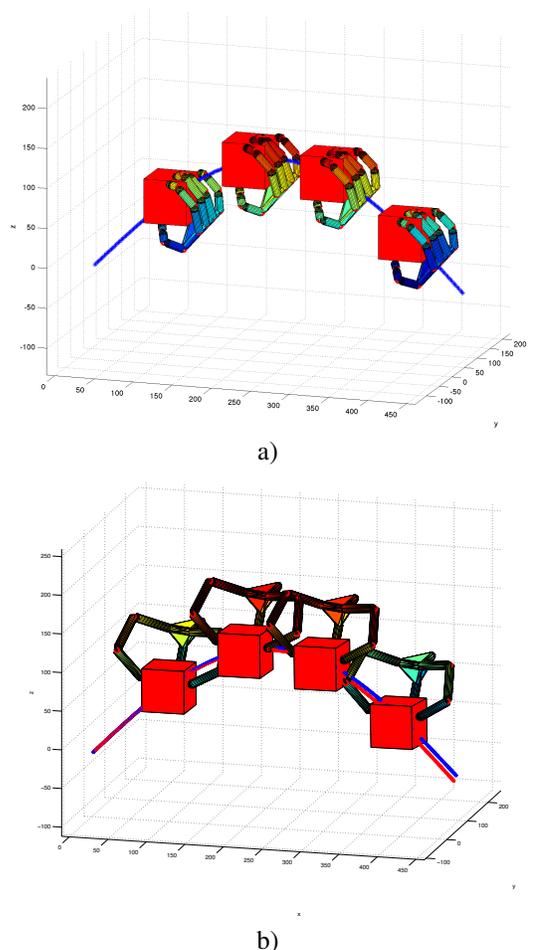


Fig. 5: Mapping human hand motion on a robotic hand during the execution of a trajectory combined with the activation of hand synergies: a) human hand and object trajectory; b) robotic hand trajectory, the blue line represents human hand object center trajectory, the red one shows the obtained robotic hand object center trajectory.

stration, in which a mapping between the human hand and robotic hands is necessary. The development of a mapping function between human and robotic hands, even with dissimilar kinematics, is necessary to solve these issues. In this paper we describe a mapping procedure based on the task space, whose main requirement is the definition of a series of reference points both on the human and on the robotic hand. When the human hand changes its joint configurations, the displacement of its reference points is used to define a homogeneous matrix able to capture the hand motion. The same homogeneous transformation matrix is adopted to evaluate the displacements of the robotic hand reference points, and consequently, through inverse kinematics techniques, the robotic hand joint variations.

The advantages of this type of mapping is that it does not requires empirical and heuristic considerations, it is general and can be applied to robotic hands with a kinematic structure very different from the anthropomorphic one. The mapping function is nonlinear, and depends on the initial

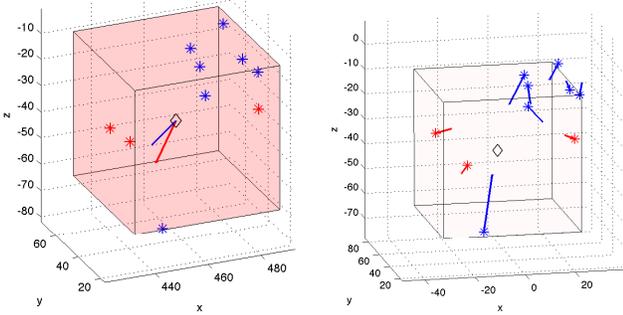


Fig. 6: a) In-hand object center motion direction: human hand (blue arrow) and robotic hand (red arrow). b) contact force variations in the human hand contact points (blue arrow) and in the robotic hand contact points (red arrows)

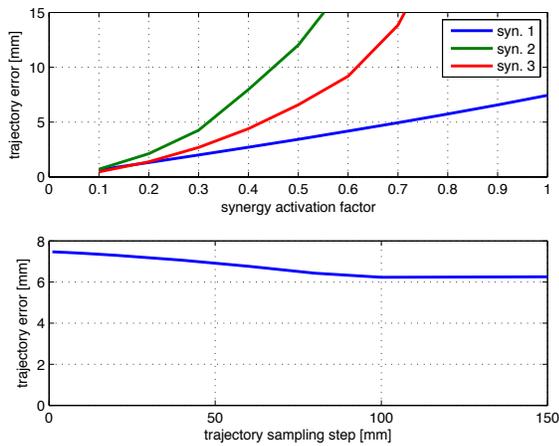


Fig. 7: Trajectory error sensitivities. Upper diagram: sensitivity of the trajectory error on the coefficient of synergy activation, for the first three synergies. Lower diagram: sensitivity on the trajectory discretization step.

reference configurations of human and robotic hand.

This is a preliminary presentation of a homogeneous transformation based mapping procedure. The numerical tests presented in the paper show that its performance depends on relative configuration of the hands and this aspect needs to be furthermore analyzed and compared with other mapping methods. However, since the method is based on the transformation of a virtual set of points in the task space, we expect that the dependency of mapping performance on the relative configurations between the hands is substantially due to the kinematic limits of the robotic hand and not on the mapping features, like, for instance, in the joint-to-joint mapping method. Other parameters have to be considered in the analysis, for instance the role of the number and location of the reference points on both the hands.

REFERENCES

- [1] A. Bicchi, "Hands for dextrous manipulation and robust grasping: a difficult road towards simplicity," *IEEE Trans. on Robotics and Automation*, vol. 16, pp. 652–662, December 2000.
- [2] A. Peer, S. Eidenkel, and M. Buss, "Multi-fingered telemanipulation - mapping of a human hand to a three finger gripper," in *Robot and Human Interactive Communication, 2008. RO-MAN 2008. The 17th IEEE International Symposium on*, pp. 465–470, August 2008.
- [3] N. Y. Lii, Z. Chen, M. A. Roa, A. Maier, B. Pleintinger, and C. Borst, "Toward a task space framework for gesture commanded telemanipulation," in *RO-MAN, 2012 IEEE*, pp. 925–932, IEEE, 2012.
- [4] M. T. Ciocarlie and P. K. Allen, "Hand posture subspaces for dexterous robotic grasping," *The International Journal of Robotics Research*, vol. 28, pp. 851–867, July 2009.
- [5] G. Gioioso, G. Salvietti, M. Malvezzi, and D. Prattichizzo, "Mapping synergies from human to robotic hands with dissimilar kinematics: an approach in the object domain," *IEEE Trans. on Robotics*, vol. 29, no. 4, pp. 825–837, 2013.
- [6] G. Gioioso, G. Salvietti, M. Malvezzi, and D. Prattichizzo, "An object-based approach to map human hand synergies onto robotic hands with dissimilar kinematics," in *Robotics: Science and Systems VIII*, Sidney, Australia: The MIT Press, July 2012.
- [7] M. Gabbicini, A. Bicchi, D. Prattichizzo, and M. Malvezzi, "On the role of hand synergies in the optimal choice of grasping forces," *Autonomous Robots*, pp. 1–18.
- [8] G. Salvietti, G. Gioioso, M. Malvezzi, D. Prattichizzo, A. Serio, E. Farnioli, M. Gabbicini, A. Bicchi, I. Sarakoglou, N. Tsagarakis, and D. Caldwell, "Hands.dvi: A device-independent programming and control framework for robotic hands," in *Gearing up and accelerating cross-fertilization between academic and industrial robotics research in Europe - Technology transfer experiments from the ECHORD project*, pp. 197–215, Springer Tracts in Advanced Robotics, Springer, 2014.
- [9] A. Bicchi, "On the closure properties of robotic grasping," *The Int. J. of Robotics Research*, vol. 14, no. 4, pp. 319–334, 1995.
- [10] G. Salvietti, L. Meli, G. Gioioso, M. Malvezzi, and D. Prattichizzo, "Object-based bilateral telemanipulation between dissimilar kinematic structures," in *Proc. IEEE/RSJ Int. Symp. Intelligent Robots and Systems*, (Tokyo, Japan), 2013.
- [11] K. Shoemake and T. Duff, "Matrix animation and polar decomposition," in *Proceedings of the conference on Graphics interface*, vol. 92, pp. 258–264, Citeseer, 1992.
- [12] D. Guan-Suo, "Determination of the rotation tensor in the polar decomposition," *Journal of elasticity*, vol. 50, no. 3, pp. 197–207, 1998.
- [13] A. Hoger and D. E. Carlson, "Determination of the stretch and rotation in the polar decomposition of the deformation gradient," *Quarterly of applied mathematics*, vol. 42, no. 1, pp. 113–117, 1984.
- [14] R. Murray, Z. Li, and S. Sastry, *A mathematical introduction to Robotic Manipulation*. 1994.
- [15] R. Goldman, *Recovering the Data from the Transformation Matrix*, vol. VII-2 of *GEMS*. Academic Press, 1991.
- [16] N. J. Higham and R. S. Schreiber, "Fast polar decomposition of an arbitrary matrix," *SIAM Journal on Scientific and Statistical Computing*, vol. 11, no. 4, pp. 648–655, 1990.
- [17] S. Mulatto, A. Formaglio, M. Malvezzi, and D. Prattichizzo, "Animating a deformable hand avatar with postural synergies for haptic grasping," in *Haptics: Generating and Perceiving Tangible Sensations. Eurohaptics 2010*, Lecture Notes in Computer Science, pp. 203–210, Amsterdam, The Netherlands: Springer Verlag, 2010.
- [18] D. Prattichizzo and J. Trinkle, "Grasping," in *Handbook on Robotics* (B. Siciliano and O. Kathib, eds.), pp. 671–700, Springer, 2008.
- [19] M. Malvezzi, G. Gioioso, G. Salvietti, D. Prattichizzo, and A. Bicchi, "Syngrasp: a matlab toolbox for grasp analysis of human and robotic hands," in *Proc. IEEE Int. Conf. on Robotics and Automation*, 2013.
- [20] M. Santello, M. Flanders, and J. F. Soechting, "Postural hand synergies for tool use," *The Journal of Neuroscience*, vol. 18, pp. 10105–10115, December 1998.
- [21] D. Prattichizzo, M. Malvezzi, and A. Bicchi, "On motion and force controllability of grasping hands with postural synergies," in *Robotics: Science and Systems VI*, pp. 49–56, Zaragoza, Spain: The MIT Press, June 2011.
- [22] D. Prattichizzo, M. Malvezzi, M. Gabbicini, and A. Bicchi, "On motion and force controllability of precision grasps with hands actuated by soft synergies," *IEEE Transactions on Robotics*, vol. in press, pp. 1–17, 2013.
- [23] A. T. Miller, S. Knoop, H. I. Christensen, and P. K. Allen, "Automatic grasp planning using shape primitives," in *Robotics and Automation, 2003. Proceedings. ICRA'03. IEEE International Conference on*, vol. 2, pp. 1824–1829, IEEE, 2003.